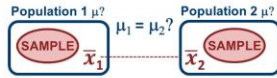


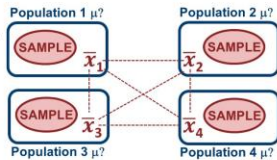
# The one factor ANOVA: introduction

## Testing population means

Two populations, one t-test.  
 ► ~28% chance type I error.



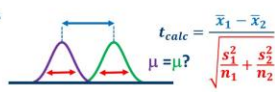
Four populations, six t-tests.  
 ► ~28% chance type I error.



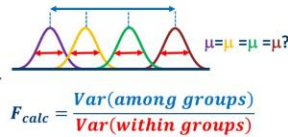
Multiple comparisons inflate the overall probability of type I error. We need a new test with 5% overall probability of type I error.

## Testing population means

**The t-test:** we compare the difference to the combined standard error.



**The ANOVA:** we compare the variance of the means to the total variance within the groups (with an F-test).



## The ANOVA

Conceptual hypotheses:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$$

$$H_A: \text{at least two means differ}$$

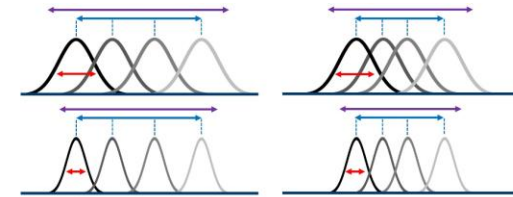
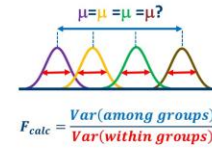
Formal hypotheses:

$$H_0: MSA \leq MSW$$

$$H_A: MSA > MSW$$

(Note: "mean sums" MSA and MSW, are variances)

Result: 1+ means differ, which ones?



All data, **Sum of Squares Total, SST**  
 Among groups, **Sum of Squares Among, SSA** **SST = SSA + SSW**  
 Within groups, **Sum of Squares within, SSW**

**Sum of Squares Total, SST**  
**Sum of Squares Among, SSA**  
**Sum of Squares Within, SSW**  
**SST = SSA + SSW**

$$MSA = \frac{SSA}{df_{among}}$$

$$MSW = \frac{SSW}{df_{within}}$$

$$F_{calc} = \frac{Var(among\ groups)}{Var(within\ groups)} = \frac{MSA}{MSW}$$

## Degrees of freedom

df for among groups:  $k - 1$   
 df for within groups:  $k(n - 1) = N - k$   
 df for entire data set:  $N - 1$

$$MSA = \frac{SSA}{k - 1}$$

$$MSW = \frac{SSW}{N - k}$$

$$F_{calc} = \frac{MSA}{MSW}$$

## The ANOVA is homoscedastic

The ANOVA requires equal variances to prevent one **unusually variable group** from overwhelming the SSW value.

Unequal variances can cause type II errors (i.e., failure to detect differences between population means).

- A prerequisite is therefore a test for equal variances (e.g.,  $F_{max}$  test).
- If the variances are equal, then we can do ANOVA.
- If the variances are not equal, then we can't and have two options: transform data or use a different test (e.g., Kruskal-Wallis).

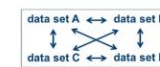
## Option 1: Bonferroni corrected t-tests

Go back to data sets and do all pairwise t-tests, but with a smaller  $\alpha$  value (i.e., less than 0.05) as the threshold for significance. This is the **Bonferroni correction** for the critical alpha value.

$$\text{Use } \alpha^* = \frac{0.05}{n} \text{ where } n \text{ is the number of t-tests.}$$



Ex: Use  $\alpha = 0.05/3 = 0.01666$  if doing 3 comparisons.



Ex: Use  $\alpha = 0.05/6 = 0.00833$  if doing 6 comparisons.

## How do we get SST, SSA, and SSW ?

**SST:** calculate sum of squares for all data values (comparing to overall mean).

This measures the overall variation. How the data values differ depending on which groups they're in, combined with the noise within each group.



## The ANOVA Table

Data is usually presented in an ANOVA Table

Source	df	SS	MS	F	p
Among groups	$k - 1$	SSA	$MSA = \frac{SSA}{k-1}$	$F = \frac{MSA}{MSW}$	?
Within groups	$N - k$	SSW	$MSW = \frac{SSW}{N-k}$		
Total	$N - 1$				

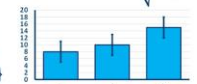
Indicates if any means are significantly different

## Option 2: Tukey-Cramer comparison intervals

Calculate the **MSD (minimum significant difference)** AKA the **HSD (honestly significant difference)** and create intervals around each sample mean of 1/2 MSD. Non-overlapping intervals indicate differing means.

$$MSD = HSD = Q_{\alpha, k, df=N-k} \times \sqrt{\frac{MSW}{n}}$$

e.g., if MSD = 6  
 $\bar{x}_A = 8 \rightarrow \{5, 11\}$   
 $\bar{x}_B = 10 \rightarrow \{7, 13\}$   
 $\bar{x}_C = 15 \rightarrow \{12, 18\}$



## How do we get SST, SSA, and SSW ?

**SSA:** calculate sum of squares of the  $\bar{x}_i$  values (comparing to overall mean), multiplying by the group sample size,  $n$ .

This measures the variation that is associated with the differences in the means of the groups. How much is the data spread out because the means of the groups are spread out?

## How do we get SST, SSA, and SSW ?

**SSW:** calculate the sum of squares values separately for each of the  $k$  groups using the group means and sum them.

This measures how much of the variation comes from variation within each of the groups. How much noise is in the data?

Problem: Type I error high when doing multiple comparisons

Solution: **AN**alysis **O**f **V**ariance = **ANOVA**

Prerequisite = test for equality of population variances  
 Do the ANOVA

Conceptual hypotheses:  $H_0: \mu_1 = \mu_2 = \dots = \mu_k$   
 $H_A: \text{at least two differ}$   
 Formal hypotheses:  $H_0: MSA \leq MSW$   
 $H_A: MSA > MSW$

Calculate SST, SSA, SSW, MSA, MSW

Calculate  $F = MSA/MSW$

Create ANOVA table

Determine p value and "reject" or "fail to reject"  $H_0$

If  $H_0$  rejected: 1. Perform Bonferroni corrected t-tests  
 2. Calculate MSD (i.e., HSD) and compare

