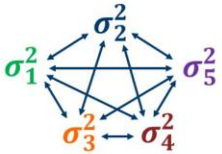
## Comparing population variances

We want to know if the variances of more than two populations differ. $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = \sigma_5^2$?

If we do all the possible pairwise comparisons, the risk of type I error is too high. (probability of type I is $\alpha$ per test)

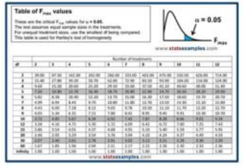



10 x 0.05 = 0.50

We therefore use a test which compares the largest variance to the smallest to see if at least one pair of variances differ.

## The variance ratio F test formal procedure

- Create null and alternative hypotheses:
  $H_0: \sigma_1^2 = \cdots = \sigma_{n-1}^2 = \sigma_n^2$
  $H_A$: *at least one is different*
- Calculate $F_{MAX}$ value $F_{MAX} = \frac{s^2_{largest}}{s^2_{smallest}}$
- Compare $F_{MAX}$ to various $F_{MAX}$ values ($\alpha$=0.05 or $\alpha$= 0.01)
- Determine probability, **p value**, of seeing $F_{MAX}$ as large as we do.
- Decide to "reject $H_0$" or "fail to reject $H_0$" based on the p value.
  $H_0: \sigma_1^2 = \cdots = \sigma_{n-1}^2 = \sigma_n^2$ ← non-small p values.
  $H_A$: *at least one different* ← small p values.




Columns: number of groups
Rows: df per group

## Example #1

Consider four samples from populations:

| | | | | | | | Mean | Var |
|---|---|---|---|---|---|---|---|---|
| Sample 1: | 12 | 14 | 16 | 14 | 11 | 17 | 14 | 5.20 |
| Sample 2: | 21 | 23 | 25 | 22 | 22 | 19 | 22 | 4.00 |
| Sample 3: | 6 | 7 | 8 | 9 | 9 | 15 | 9 | 14.00 |
| Sample 4: | 5 | 1 | 7 | 5 | 4 | 8 | 5 | 6.00 |

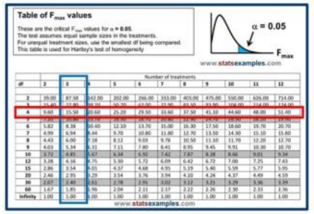
Are any of the population variances different from one another?
With what degree of confidence do we make this conclusion?

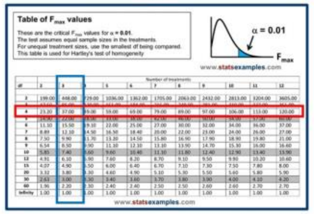
$$F_{MAX} = \frac{s^2_{largest}}{s^2_{smallest}} = \frac{14.00}{4.00} = 3.50$$

## Example #2 $F_{MAX}$ = 17.08

Are any of the population variances different from one another?
With what degree of confidence do we make this conclusion?

***groups = 3*** $F_{MAX:\alpha=0.05,3,4} = 15.50$
***$df_{group}$ = 5-1=4*** $F_{MAX:\alpha=0.01,3,4} = 37.00$




"At least one of these variances is *significantly different* from at least one of the others ( 0.01 < p < 0.05 )."

## Example #2

Consider three samples from populations:

| | | | | | | Mean | Var |
|---|---|---|---|---|---|---|---|
| Sample 1: | 16 | 19 | 22 | 21 | 22 | 20 | 6.50 |
| Sample 2: | 15 | 13 | 18 | 19 | 15 | 16 | 6.00 |
| Sample 3: | 14 | 15 | 16 | 13 | 37 | 19 | 102.50 |

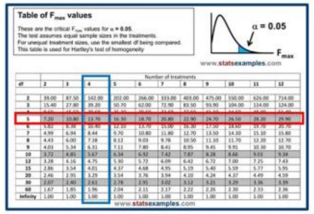
Are any of the population variances different from one another?
With what degree of confidence do we make this conclusion?

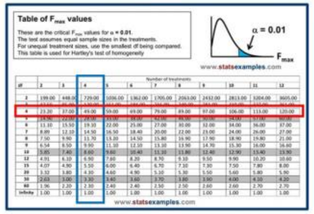
$$F_{MAX} = \frac{s^2_{largest}}{s^2_{smallest}} = \frac{102.50}{6.00} = 17.08$$

## Example #1 $F_{MAX}$ = 3.50

Are any of the population variances different from one another?
With what degree of confidence do we make this conclusion?

***groups = 4*** $F_{MAX:\alpha=0.05,4,5} = 13.70$
***$df_{group}$ = 6-1=5*** $F_{MAX:\alpha=0.01,4,5} = 49.00$




"The variances of these populations are *not significantly different* from one another ( p > 0.05 )."

## Example #2 (transformed values)

Transformed samples from populations:

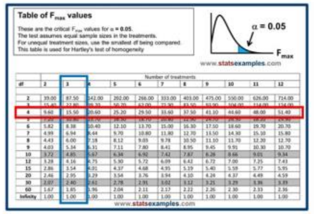
| | | | | | | Mean | Var |
|---|---|---|---|---|---|---|---|
| Sample 1: | 4.0000 | 4.3589 | 4.6904 | 4.5826 | 4.6904 | 4.464 | 0.0857 |
| Sample 2: | 3.8730 | 3.6056 | 4.2426 | 4.3589 | 3.8730 | 3.991 | 0.0938 |
| Sample 3: | 3.7417 | 3.8730 | 4.0000 | 3.6056 | 6.0828 | 4.261 | 1.0592 |

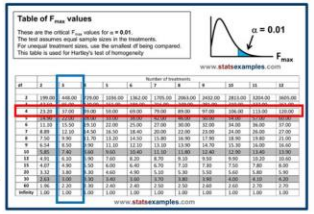
$$F_{MAX} = \frac{s^2_{largest}}{s^2_{smallest}} = \frac{1.0592}{0.0857} = 12.35$$

## Example #2 (transformed values) $F_{MAX}$ = 12.35

Are any of the transformed population variances different from one another?
With what degree of confidence do we make this conclusion?

***groups = 3*** $F_{MAX:\alpha=0.05,3,4} = 15.50$
***$df_{group}$ = 5-1=4*** $F_{MAX:\alpha=0.01,3,4} = 37.00$




"The variances of these transformed populations are *not significantly different* from one another ( p > 0.05 )."

## Caution about the $F_{MAX}$ test

A strong assumption of the $F_{MAX}$ test is normal population distributions.

For example, if you look at the distributions of the example data sets used in the video, they aren't normal.

We prefer a Levene's, Bartlett's, or Brown-Forsythe test (math is more complicated).

If we transform the data, our subsequent tests apply to the transformed values, not the originals.