BINOMIAL PROBABILITY

STEP-BY-STEP EXAMPLES

Let's do some examples

$p(k) = \binom{n}{k} p^k (1-p)^{(n-k)}$

The basic scenario

p=prob(success)

1-p=prob(failure)

We classify some events as successes and the rest as failures (an arbitrary label).
Trial: an observation of an event
Success: an outcome that fits our criterion

The probability of seeing **x** successes when we do **n** trials.

$$P(x) = \binom{n}{x} p^x (1-p)^{(n-x)} = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{(n-x)}$$

Example 1

p(male)=0.5

p(female)=1-p=1-0.5

Consider a population of frogs in a lake.
Assume the sex ratio is 0.5.
Collect 6 frogs and determine the sexes.
What are the probabilities of 0 males, 1 male, 2 males, etc.?

$$P(x) = \binom{n}{x} p^x (1-p)^{(n-x)} = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{(n-x)}$$

$$P(0) = \frac{6!}{0!\,(6-0)!} 0.5^0 (1-0.5)^{(6-0)}$$

Example 1

p(male)=0.5

p(female)=1-p=1-0.5

$$P(0) = \frac{6!}{0!\,(6-0)!} 0.5^0 (1-0.5)^{(6-0)}$$

$$P(0) = \frac{6!}{0!\,6!} (1)(0.5)^6 = (1)(1)(0.5)^6 = 0.015625$$

$$P(1) = \frac{6!}{1!\,(6-1)!} 0.5^1 (1-0.5)^{(6-1)}$$

$$P(1) = \frac{6!}{1!\,5!} (0.5)(0.5)^5 = \frac{6}{1}(0.5)(0.03125) = 0.09375$$

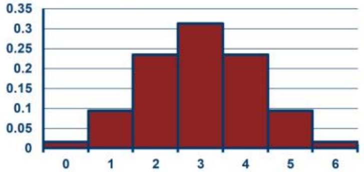
Example 1

p(male)=0.5

p(female)=1-p=1-0.5

Consider a population of frogs in a lake.
Assume the sex ratio is 0.5.
Collect 6 frogs and determine the sexes.

$P(0) = 0.015625$
$P(1) = 0.09375$
$P(2) = 0.234375$
$P(3) = 0.3125$
$P(4) = 0.234375$
$P(5) = 0.09375$
$P(6) = 0.015625$



Example 1

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Consider a population of frogs in a lake.
Assume the sex ratio is 0.5.
Collect 6 frogs and determine the sexes.
What are the probabilities of 0 males, 1 males, 2 males, etc.?

$P(0) = 0.015625$ $P(1) = 0.09375$

$$P(2) = \binom{6}{2}(0.5)^2(1-0.5)^{(6-2)} = \frac{6!}{2!\,(6-2)!}(0.5)^2(1-0.5)^{(6-2)}$$

$$P(2) = \binom{6}{2} = \frac{6!}{2!\,4!}(0.25)(0.5)^4 = \frac{6\times 5}{2\times 1}(0.25)(0.0625) = 0.234375$$

CALCULATING FRACTIONS WITH FACTORIALS

$$\frac{20!}{7!\,(20-7)!} = \frac{20!}{7!\,13!} = \frac{20\times 19\times 18\times 17\times 16\times 15\times 14\times 13!}{7!\,13!}$$

$$= \frac{20\times 19\times 18\times 17\times 16\times 15\times 14}{7!}$$

$$= \frac{20\times 19\times 18\times 17\times 16\times 15\times 14}{7\times 6\times 5\times 4\times 3\times 2\times 1!}$$

$$= \frac{20\times 19\times 3\times 17\times 4}{1} = 77{,}250$$

CALCULATING FRACTIONS WITH FACTORIALS

$6! = 6\times 5\times 4\times 3\times 2\times 1 = 6\times 5!$
$6! = 6\times 5\times 4\times 3\times 2\times 1 = 6\times 5\times 4!$
$6! = 6\times 5\times 4\times 3\times 2\times 1 = 6\times 5\times 4\times 3!$

$$\frac{6!}{1!\,(6-1)!} = \frac{6!}{5!} = \frac{6\times 5!}{5!} = \frac{6}{1}$$

$$\frac{n!}{x!\,(n-x)!} = \frac{n\times(n-1)\ldots(n-x+1)\times(n-x)!}{x!\,(n-x)!} = \frac{n\times(n-1)\ldots(n-x+1)}{x!}$$

$$\frac{20!}{7!\,(20-7)!} = ?$$

Example 2

Consider a roulette wheel.
There are 35 pockets: 18 even (8 **red**, 10 **black**), 18 odd (10 **red**, 8 **black**), 1 zero (**green "0"**)
If we spin the wheel 3 times, what are the probabilities of getting 0, 1, 2, or 3 evens?

$$P(x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{(n-x)}$$

$$P(x) = \frac{3!}{x!\,(3-x)!} p^x (1-p)^{(3-x)}$$

Example 2

There are 35 pockets: 18 even (8 **red**, 10 **black**), 18 odd (10 **red**, 8 **black**), 1 zero (**green "0"**)
What are the probabilities of getting 0, 1, 2, or 3 evens?
First, what is p?

$$p = \frac{18}{18+18+1} = \frac{18}{37} = 0.486486$$

$$P(x) = \frac{3!}{x!\,(3-x)!} p^x (1-p)^{(3-x)}$$

$$P(x) = \frac{3!}{x!\,(3-x)!} (0.486486)^x (0.513514)^{(3-x)}$$

Example 2

$p = 0.486486$

What are the probabilities of getting **0**, 1, 2, or 3 evens?

$$P(x) = \frac{3!}{x!\,(3-x)!} (0.486486)^x (0.513514)^{(3-x)}$$

$$P(0) = \frac{3!}{0!\,(3-0)!} (0.486486)^0 (0.513514)^{(3-0)}$$

$$P(0) = \frac{3!}{(1)3!}(1)(0.513514)^3 = (0.513514)^3 = 0.1354$$

Example 2

$p = 0.486486$

What are the probabilities of getting 0, **1**, 2, or 3 evens?

$$P(x) = \frac{3!}{x!\,(3-x)!} (0.486486)^x (0.513514)^{(3-x)}$$

$$P(1) = \frac{3!}{1!\,(3-1)!} (0.486486)^1 (0.513514)^{(3-1)}$$

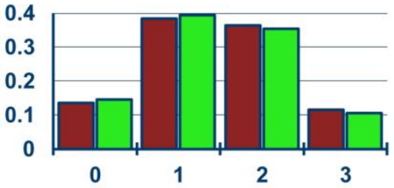
$$P(1) = \frac{3!}{(1)2!} (0.486486)(0.513514)^2$$

$$= \frac{3}{1}(0.486486)(0.263697) = 0.3849$$

Example 2

In the US there are two green pockets ("0" and "00"), but the payouts are the same, but with p=0.473684, not 0.486486.

$P(0) = 0.1354 \text{ or } 0.1458$
$P(1) = 0.3849 \text{ or } 0.3936$
$P(2) = 0.3646 \text{ or } 0.3543$
$P(3) = 0.1151 \text{ or } 0.1063$



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If we spin the wheel 3 times, what are the probabilities of getting 0, 1, 2, or 3 evens?

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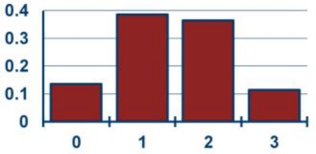
$P(2) = 0.3646$
$P(3) = 0.1151$



Example 2

$p = 0.486486$

What are the probabilities of getting 0, 1, 2, or **3** evens?

$$P(x) = \frac{3!}{x!\,(3-x)!} (0.486486)^x (0.513514)^{(3-x)}$$

$$P(3) = \frac{3!}{3!\,(3-3)!} (0.486486)^3 (0.513514)^{(3-3)}$$

$$P(3) = \frac{3!}{3!\,(1)} (0.115136)(0.513514)^0$$

$$= (1)(0.115136)(1) = 0.1151$$

Example 2

$p = 0.486486$

What are the probabilities of getting 0, 1, **2**, or 3 evens?

$$P(x) = \frac{3!}{x!\,(3-x)!} (0.486486)^x (0.513514)^{(3-x)}$$

$$P(2) = \frac{3!}{2!\,(3-2)!} (0.486486)^2 (0.513514)^{(3-2)}$$

$$P(2) = \frac{3!}{2!\,(1)} (0.236669)(0.513514)^1$$

$$= \frac{3}{1}(0.236669)(0.513514) = 0.3646$$

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