False positives and negatives

Formal statistical testing

Assuming a default of a negative result, **type I** and **type II** errors are **false positives** and **false negatives**

	Our decision: Accept H_0	Reject H_0
H_0 true	Correct	**Type I error**
H_0 false	**Type II error**	Correct

	Our test result: Negative	Positive
Negative	Correct	**False positive**
Positive	**False negative**	Correct

e.g., medical testing for cancer or HIV or whatever

H_0: person not sick
H_A: person is sick

False positives (type I errors)
- Scare the patient, cost $
False negatives (type II errors)
- person goes untreated.

	Our test result: Negative	Positive
Negative	Correct	**False positive**
Positive	**False negative**	Correct

Type II error is serious at the individual level, but relative rates of the errors change as the frequency of the condition varies.

Hypothetical test { false positive rate of 2%, false negative rate of 3%

What if we test 100,000 people and 10% have condition?

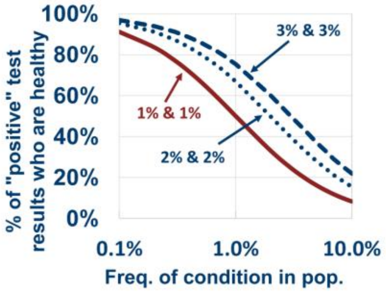
e.g., 100,000 tested	"+" test	"-" test
H_0 true: 90,000 w/o condition	1,800	88,200
H_0 false: 10,000 w/ condition	9,700	300

1,800/(1,800+9,700) = **16%** of people with "+" test are not sick.

Test "mistake" rate

If we test for rare conditions, most positive test results will be false positives.

This is true even with very good tests (e.g., 1% & 1% error)

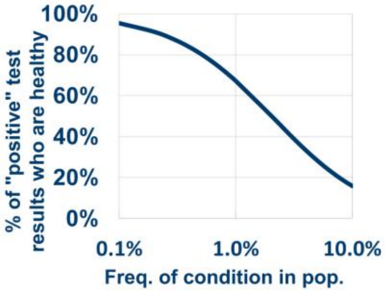


Test "mistake" rate

This figure shows the 3 values we just calculated.

The overall pattern would look like this.

Now let's consider a range of tests and frequencies.



Hypothetical test { false positive rate of 2%, false negative rate of 3%

What if we test 100,000 people and 0.1% have condition?

e.g., 100,000 tested	"+" test	"-" test
H_0 true: 99,900 w/o condition	1,998	97,902
H_0 false: 100 w/ condition	97	3

1,998/(1,998+97) = **95%** of people with "+" test are not sick.

Hypothetical test { false positive rate of 2%, false negative rate of 3%

What if we test 100,000 people and 1% have condition?

e.g., 100,000 tested	"+" test	"-" test
H_0 true: 99,000 w/o condition	1,980	97,020
H_0 false: 1,000 w/ condition	970	30

1,980/(1,980+970) = **67%** of people with "+" test are not sick.

How to balance these type I and II errors?

H_0: person not sick
H_A: person sick

	Our test result: Negative	Positive
Negative	Correct	False positive
Positive	**False negative**	Correct

Each **false negative** (**type II error**) or untested person means that they go undiagnosed until their condition worsens. This costs lives.

Solution ... **more testing?**

How to balance these type I and II errors?

H_0: person not sick
H_A: person sick

	Our test result: Negative	Positive
Negative	Correct	**False positive**
Positive	False negative	Correct

Each **false positive** (**type I error**) costs $ for more tests, causes stress, and undermines public confidence in modern medicine. This costs lives.

Solution ... **less testing?**

This tradeoff is why the advice on mammograms and prostate exams seems to keep changing.

Doing statistics is easier that applying statistics

There is no perfect answer to the tradeoff.

- ►More testing has a downside.
- ►Less testing has a downside.
- ►Both downsides cost $ and lives.

No test is perfect, and resources are limited.
We can't afford to test everyone multiple times to eliminate all initial false positives for rare conditions.
Therefore we don't test, and some people die.

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