SE

STEP-BY-STEP EXAMPLES

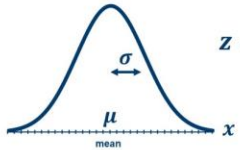
# NORMAL DISTRIBUTION

Let's do some examples

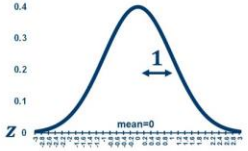

## Review: normal (Gaussian) distribution

Watch our intro to the normal distribution video for more about this

Normal distributions with a mean μ and standard deviation σ can be transformed into the standard normal distribution with mean of zero and standard deviation of one.



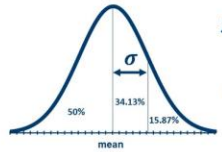

$$z = \frac{x-\mu}{\sigma}$$




## Review: normal (Gaussian) distribution

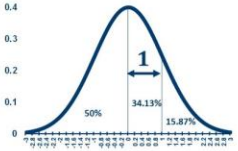
Watch our intro to the normal distribution video for more about this

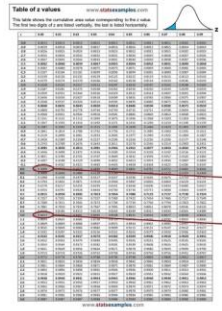
► Regions within the original normal distribution correspond to regions in the standard normal distribution.
► Areas in the standard normal distribution are known and tabulated in tables (or stored in statistics programs).



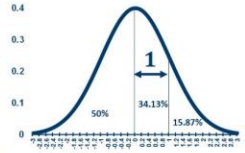

$$z = \frac{x-\mu}{\sigma}$$

Area from x = μ to x = μ + σ
same as
Area from z = 0 to z = 1



Tables (*usually*) show areas to left of Z values.




For z=0, Area to the left from table = 0.5

For z=1, Area to the left from table = 0.8413

Area between z=0 and z=1 is therefore: 0.8413 - 0.5 = 0.3413

Note, area above z=1 would be: 1 - 0.8413 = 0.1587

## Example #1 - masses of guinea pigs

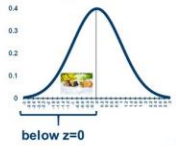
Consider a population of 80 guinea pigs in which the mean mass is 1.1 kg and the standard deviation is 200g (0.2 kg).

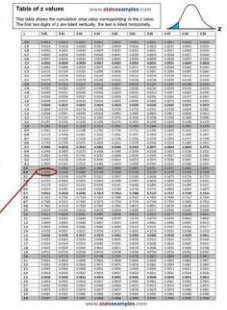
How many guinea pigs are lighter than 1.1 kg?
How many guinea pigs are lighter than 1.4 kg?
How many guinea pigs are heavier than 0.8 kg?

## Example #1 - masses of guinea pigs

How many guinea pigs are lighter than 1.1 kg?

$$z = \frac{x-\mu}{\sigma} = \frac{1.1-1.1}{0.2} = 0$$



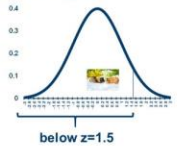
For z=0, the area to the left from table = 0.5000

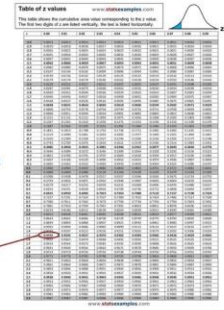
80 x 0.5000 = 40 guinea pigs

## Example #1 - masses of guinea pigs

How many guinea pigs are lighter than 1.4 kg?

$$z = \frac{x-\mu}{\sigma} = \frac{1.4-1.1}{0.2} = \frac{0.3}{0.2} = 1.5$$



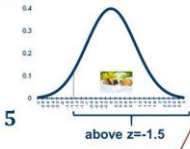
For z=1.5, the area to the left from table = 0.9332

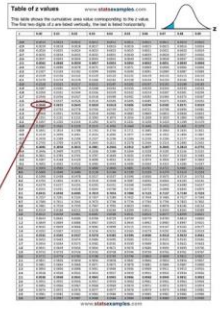
80 x 0.9332 = 74.656 guinea pigs

## Example #1 - masses of guinea pigs

How many guinea pigs are heavier than 0.8 kg?

$$z = \frac{x-\mu}{\sigma} = \frac{0.8-1.1}{0.2} = \frac{-0.3}{0.2} = -1.5$$



1 - 0.0668 = 0.9332

For z=-1.5, the area to the left from table = 0.0668

80 x 0.9332 = 74.656 guinea pigs

## Example #2 - viral load counts

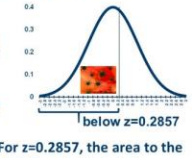
Consider a population of 15,000 infected patients with a mean viral load of 60,000/ml and standard deviation of 7,000.

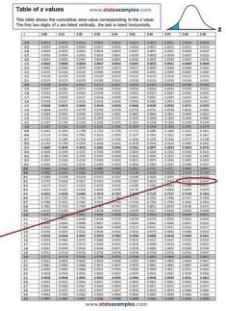
How many patients have a viral load less than 62,000?
How many patients have a viral load between 53,000 and 74,000?
What viral load value divides the population into a top 5% and bottom 95%

## Example #2 - viral load counts

How many patients have a viral load less than 62,000?

$$z = \frac{62,000 - 60,000}{7,000} = \frac{2,000}{7,000} = 0.2857$$



15,000 x 0.6103 = 9,154.5 patients
15,000 x 0.6141 = 9,211.5 patients

For z=0.2857, the area to the left from table is between 0.6103 and 0.6141

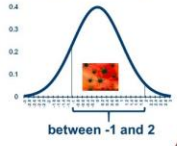
From 9,155 to 9,212 patients

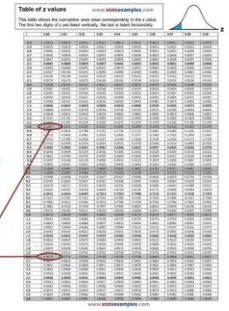
## Example #2 - viral load counts

How many patients have a viral load between 53,000 and 74,000?

$$z = \frac{53,000 - 60,000}{7,000} = \frac{-7,000}{7,000} = -1$$

$$z = \frac{74,000 - 60,000}{7,000} = \frac{14,000}{7,000} = 2$$



0.8772 - 0.1587 = 0.7185

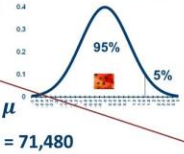
For z = -1, 0.1587 to the left

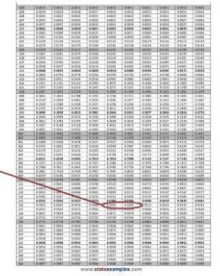
For z = 2, 0.8772 to the left

15,000 x 0. 7185 = 10,777.5 patients

What viral load value divides the population into a top 5% and bottom 95%

From areas in table
0.9495 is z= 1.64
0.9505 is z= 1.65



$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \sigma z + \mu$$

x = (7,000)(1.64) + 60,000 = 71,480
x = (7,000)(1.65) + 60,000 = 71,550
Viral load between 71,480 and 71,550

$$Interpolate: \left(\frac{1.64+1.65}{2}\right)(7,000) + 60,000 = 71,515$$

## Example #3 - exam scores

Consider a set of 2,000 exams with a mean score of 74 and a standard deviation of 6.

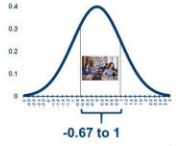
How many scores are between 70 and 80?
Which score corresponds to the 90% percentile?
What is the range for the middle 50% of the scores?

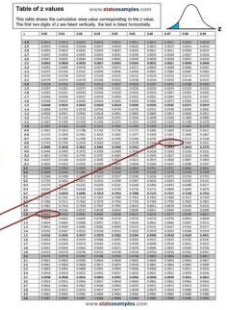
## Example #3 - exam scores

How many scores are between 70 and 80?

$$z = \frac{70-74}{6} = \frac{-4}{6} = -0.66$$

$$z = \frac{80-74}{6} = \frac{6}{6} = 1$$



0.8413 - 0.2514 = 0.5899

For z = -0.66, 0.2514 to the left

For z = 1, 0.8413 to the left

2,000 x 0. 5899 = 1,179.8 scores

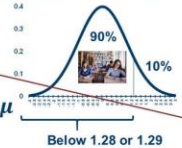
## Example #3 - exam scores

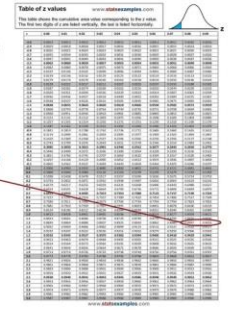
Which score corresponds to the 90% percentile?

From areas in table
0.8997 is z= 1.28
0.9015 is z= 1.29

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \sigma z + \mu$$

x = (6)(1.28) + 74 = 81.68
x = (6)(1.29) + 74 = 81.74
A score between 81.68 and 81.74



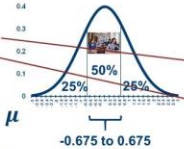



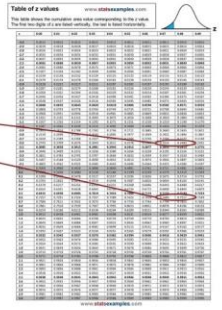
$$Interpolate: \left(\frac{15}{18}\right)(81.68) + \left(\frac{3}{18}\right)(81.74) = 81.69$$

## Example #3 - exam scores

What is the range for the middle 50% of the scores?

From areas in table
0.2514/0.2483 is z= -0.67/-0.68
0.7486/0.7517 is z= 0.67/0.68



$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \sigma z + \mu$$

x = (6)(-0.675) + 74 = 69.95
x = (6)(0.675) + 74 = 78.05
IQR = 8.1

Between 69.95 and 78.05

# StatsExamples.com