

ONE SAMPLE T TEST

STEP-BY-STEP EXAMPLES

Let's do some examples

Review: one-sample t-test

We want to know if the population mean is μ_0 . We can't measure the population mean. We calculate a mean and confidence interval. Does the confidence interval include μ_0 ?

POPULATION $\mu = \mu_0$?
SAMPLE \bar{x}_1

95% CI Reasonable probability that $\mu = \mu_0$

95% CI Very low probability that $\mu = \mu_0$

Watch our intro to the one sample t-test video for more about this!

Review: one-sample t-test

We don't do: We compare half the width of the CI to the distance between the sample mean and μ_0

Distance between sample mean and μ_0 $>$ Half full width of 95% confidence interval

$\frac{\bar{x} - \mu_0}{SE} > t_{\alpha=0.025}$

$\frac{\bar{x} - \mu_0}{SE} > t_{\alpha=0.025}$

"t_{calc}" > "t_{crit}"?

Review: one-sample t-test

Create a null hypothesis and alternative hypothesis:
 $H_0: \mu = \mu_0$
 $H_A: \mu \neq \mu_0$

Calculate $t_{calc} = \frac{\bar{x} - \mu_0}{SE}$

Compare t_{calc} to various t_{crit} values (i.e., widths of CIs).

Determine probability, **p value**, of seeing t_{calc} as extreme as we do.

Decide to "reject H_0 " or "fail to reject H_0 " based on the p value.
 $H_0: \mu = \mu_0$ consistent with non-small p values.
 $H_A: \mu \neq \mu_0$ would give us small p values.

Example #2 - heights of birds

Is the mean of the population they come from equal to 1 meter?
With what degree of confidence do we make this conclusion?

$t_{calc} = \frac{\bar{x} - \mu_0}{SE} = \frac{1.11 - 1}{0.056490} = 1.9473$

$df = 10 - 1 = 9 \rightarrow t_{0.025,9} = 2.262$

95% CI $t_{calc} > t_{crit}$?
 $1.9473 > 2.262$?

"The population mean is not significantly different from 1 meter"

Example #2 - heights of birds

Consider a sample of 10 penguin heights:
0.93, 1.18, 1.34, 1.21, 1.24, 0.97, 0.93, 1.17, 1.30, 0.83

Is the mean of the population they come from equal to 1.0 meters?
With what degree of confidence do we make this conclusion?

First step, calculate sample mean and standard error

$t_{calc} = \frac{\bar{x} - \mu_0}{SE} = \frac{1.110 - 1.0}{0.178637} = 0.6158$

$df = 10 - 1 = 9 \rightarrow t_{0.025,9} = 2.262$

"The population mean is significantly larger than 1.0m (0.025 < p < 0.04)"

Example #1 - masses of frogs

Is the mean of the population they come from equal to 24g?
With what degree of confidence do we make this conclusion?

$t_{calc} = \frac{\bar{x} - \mu_0}{SE} = \frac{26 - 24}{0.767523} = 2.6058$

$df = 12 - 1 = 11 \rightarrow t_{0.025,11} = 2.201$

95% CI $t_{calc} > t_{crit}$?
 $2.6058 > 2.201$?

"The population mean is significantly larger than 24g"

Example #1 - masses of frogs

Consider a sample of 12 frog masses:
24.0, 25.0, 29.0, 27.0, 23.0, 22.5, 24.3, 28.7, 23.8, 29.2, 30.0, 25.5

Is the mean of the population they come from equal to 24g?
With what degree of confidence do we make this conclusion?

First step, calculate sample mean and standard error

$t_{calc} = \frac{\bar{x} - \mu_0}{SE} = \frac{26.0000 - 24.0}{2.658776} = 0.7523$

$df = 12 - 1 = 11 \rightarrow t_{0.025,11} = 2.201$

"The population mean is significantly larger than 24g"

Example #2 - heights of birds

Is the mean of the population they come from equal to 1 meter?
With what degree of confidence do we make this conclusion?

$t_{calc} = 1.9473$
 $\bar{x} = 1.11$

95% CI $t_{calc} > t_{crit}$?
 $1.9473 > 2.262$?

"The population mean is not significantly different from 1 meter (0.05 < p < 0.1)"

Example #3 - clutch size

Consider a sample of 14 clutch sizes:
8, 9, 8, 5, 4, 7, 7, 8, 9, 10, 7, 5, 10, 7

Is the mean clutch size for these ducks equal to 9?
With what degree of confidence do we make this conclusion?

First step, calculate sample mean and standard error

$t_{calc} = \frac{\bar{x} - \mu_0}{SE} = \frac{7.429 - 9}{1.827747} = -3.2169$

$df = 14 - 1 = 13 \rightarrow t_{0.025,13} = 2.160$

95% CI $-t_{calc} > -t_{crit}$?
 $-3.2169 < -2.160$?

"The population mean is significantly less than 9 eggs"

Example #3 - clutch size

Is the mean clutch size for these ducks equal to 9?
With what degree of confidence do we make this conclusion?

$t_{calc} = -3.2169$
 $\bar{x} = 7.429$

95% CI $-t_{calc} > -t_{crit}$?
 $-3.2169 < -2.160$?

"The population mean is significantly less than 9 eggs"

Example #3 - clutch size

Is the mean clutch size for these ducks equal to 9?
With what degree of confidence do we make this conclusion?

$t_{calc} = -3.2169$
 $\bar{x} = 7.429$

95% CI $-t_{calc} > -t_{crit}$?
 $-3.2169 < -2.160$?

"The population mean is significantly less than 9 eggs (0.005 < p < 0.01)"

One-tailed t-tests

Different null & alternative hypotheses:
 $H_0: \mu \geq \mu_0$ or $H_0: \mu \leq \mu_0$
 $H_A: \mu < \mu_0$ or $H_A: \mu > \mu_0$

$t_{calc} = ?$

One looks at if t_{calc} is more extreme than t_{crit} in one direction

Example #5 - parasite load

Is the mean parasite load of the population more than 10?
With what degree of confidence do we make this conclusion?

$t_{calc} = \frac{12.5 - 10}{1.101946} = 2.2687$

$df = 8 - 1 = 7 \rightarrow t_{0.05,7} = 1.895$

95% CI $t_{calc} > t_{crit}$?
 $2.2687 > 1.895$?

"The mean number of parasites is significantly larger than 10"

Example #5 - parasite load

Consider a sample of 8 parasite loads:
11, 12, 10, 14, 12, 8, 18, 15

Is the mean parasite load of the population more than 10?
With what degree of confidence do we make this conclusion?

First step, calculate sample mean and standard error

$t_{calc} = \frac{\bar{x} - \mu_0}{SE} = \frac{12.5 - 10}{1.101946} = 2.2687$

$df = 8 - 1 = 7 \rightarrow t_{0.05,7} = 1.895$

"The population mean is not significantly smaller than 10 (0.05 < p < 0.1)"

Example #4 - cholesterol values

Is the mean of the population they come from less than 200?
With what degree of confidence do we make this conclusion?

$t_{calc} = \frac{197.6 - 200}{1.386633} = -1.7312$

$df = 15 - 1 = 14 \rightarrow t_{0.05,14} = 1.761$

95% CI $t_{calc} > t_{crit}$?
 $-1.7312 > 1.761$?

"The population mean is not significantly less than 200"

Example #4 - cholesterol values

Is the mean of the population they come from less than 200?
With what degree of confidence do we make this conclusion?

$t_{calc} = -1.7312$
 $\bar{x} = 197.6$

95% CI $t_{calc} > t_{crit}$?
 $-1.7312 > 1.761$?

"The population mean is not significantly less than 200"

Example #4 - cholesterol values

Consider a sample of 15 cholesterol values:
199, 193, 195, 204, 194, 203, 198, 195, 206, 203, 205, 190, 193, 195, 191

Is the mean cholesterol value less than 200?
With what degree of confidence do we make this conclusion?

First step, calculate sample mean and standard error

$t_{calc} = \frac{\bar{x} - \mu_0}{SE} = \frac{197.6 - 200}{5.369224} = -0.4656$

$df = 15 - 1 = 14 \rightarrow t_{0.05,14} = 1.761$

"The population mean is significantly larger than 200"

Example #5 - parasite load

Is the mean parasite load of the population more than 10?
With what degree of confidence do we make this conclusion?

$t_{calc} = 2.2687$
 $\bar{x} = 12.5$

95% CI $t_{calc} > t_{crit}$?
 $2.2687 > 1.895$?

"The mean number of parasites is significantly larger than 10 (0.025 < p < 0.05)"

Note that this was only significant because we used $\alpha = 0.05$ instead of $\alpha = 0.025$.

$t_{calc} = 2.2687$
 $\bar{x} = 12.5$

95% CI $t_{calc} > t_{crit}$?
 $2.2687 > 1.895$?

If we had calculated the sample mean first, and then decided on the direction, we would really be using the outer 10% of the area.

Caution: beware of one-tailed t-tests

Quick example #6

If a hypothesized pop. mean = 25.
Sample $\bar{x} = 27, s = 6.35, n = 40$.

$SE = \frac{6.35}{\sqrt{40}} = 1.004023$ $df = 39(35)$

$t_{calc} = \frac{27 - 25}{1.004023} = 1.99198$

$t_{calc} > t_{crit}$? $1.99 > 2.030$?

"The population mean is not significantly different from 25 (0.05 < p < 0.1)"

Quick example #7

If a hypothesized pop. mean = 25.
Sample $\bar{x} = 27, s = 6.35, n = 68$.

$SE = \frac{6.35}{\sqrt{68}} = 0.757924$ $df = 67(60)$

$t_{calc} = \frac{27 - 25}{0.757924} = 2.63878$

$t_{calc} > t_{crit}$? $2.6388 > 2.000$?

"The population mean is significantly larger than 25 (0.01 < p < 0.02)"

Caution about one sample t-tests

Technically, data should be normally distributed, but the CLT handles this for larger samples unless the data is very weird.

Always report the p-value range (or exact p-value) and direction when reporting results.

Only do a one-tailed test under 2 conditions:
1. You *only care* about one direction.
2. You have an *a priori* reason to test in only one direction. You **CANNOT** look at data to choose the direction.

Always remember the risk of a type I or type II error.