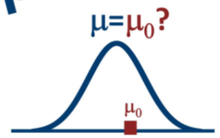
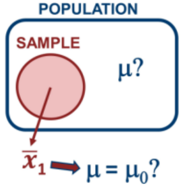
The one sample t-test

To see if the mean is what we think



Testing a population mean

We want to know if the population mean is a certain value, μ_0.
We can't measure the population.
We take a random sample.
We calculate the sample mean.



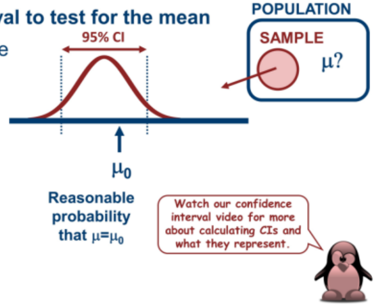
The sample mean is an estimate of the population mean, but sampling error makes it inaccurate.

Q: What is the probability the population mean is μ_0, based on the sample mean and observed variation?

Using a confidence interval to test for the mean

We compare the confidence interval (i.e., CI) to the hypothesized population mean, μ_0.

- **μ_0 inside CI** = lack of evidence that μ is different from μ_0.
- μ_0 outside CI = is evidence that μ is different from μ_0.



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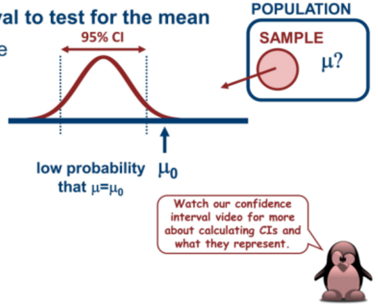
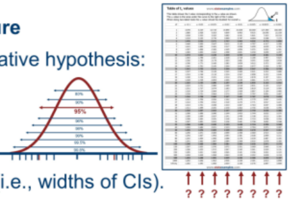
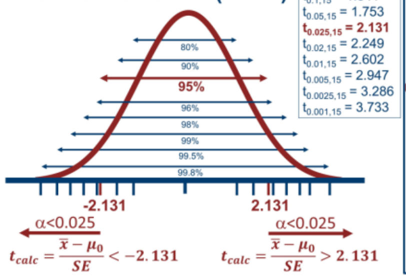


The two-tailed t test formal procedure

- Create a null hypothesis and alternative hypothesis:
  H_0: $\mu=\mu_0$
  H_A: $\mu\neq\mu_0$
- Calculate t_{calc} $\quad t_{calc}=\dfrac{\bar{x}-\mu_0}{SE}$
- Compare t_{calc} to various t_{crit} values (i.e., widths of CIs).
- Determine probability, **p value**, of seeing t_{calc} as extreme as we do.
- Decide to "reject H_0" or "fail to reject H_0" based on the p value.
  H_0: $\mu=\mu_0$ consistent with non-small p values.
  H_A: $\mu\neq\mu_0$ would give us small p values.



t test for 16 values (df=15)

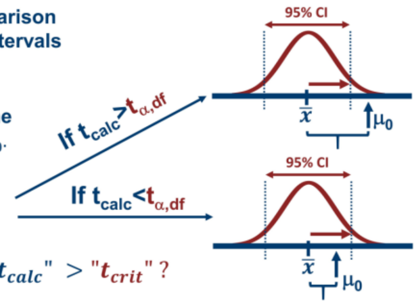


$t_{0.1,15}$ = 1.341
$t_{0.05,15}$ = 1.753
$t_{0.025,15}$ = 2.131
$t_{0.02,15}$ = 2.249
$t_{0.01,15}$ = 2.602
$t_{0.005,15}$ = 2.947
$t_{0.0025,15}$ = 3.286
$t_{0.001,15}$ = 3.733

The t test is a comparison using confidence intervals

Comparing the **width of the CI** to the **distance between the sample mean and μ_0.**

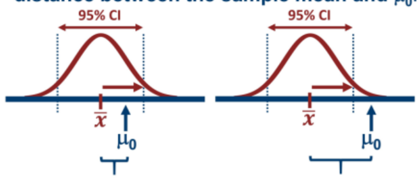
$$t_{calc}=\frac{\bar{x}-\mu_0}{SE}$$



The t test is a comparison using confidence intervals

We could calculate the confidence interval and see if includes μ_0. Instead, we calculate **the width of half the CI** and compare to the **distance between the sample mean and μ_0.**

$$Distance>\frac{1}{2}CI$$
$$Distance=\bar{x}-\mu_0$$
$$\frac{1}{2}CI=t_{\alpha,df}\times SE$$
$$\bar{x}-\mu_0>t_{\alpha,df}\times SE$$
$$\frac{\bar{x}-\mu_0}{SE}>t_{\alpha,df}$$
$$\text{"}t_{calc}\text{"}>\text{"}t_{crit}\text{"}$$



The p value

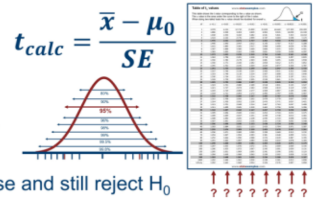
Context: H_0: $\mu=\mu_0$ and H_A: $\mu\neq\mu_0$

$$t_{calc}=\frac{\bar{x}-\mu_0}{SE}$$

- Determine probability, **p value**, of seeing t_{calc} as extreme as we do.

Technical definition: The p value is the smallest α value you could use and still reject H_0

Conceptual definition: The p value is the probability of seeing the sample data you do, if the null hypothesis is correct.



The p value

Conceptual definition: The p value is the probability of seeing the sample data you do, if the null hypothesis is correct.

is equivalent to ...

The p value is the probability of obtaining the t_{calc} statistic (or more extreme) that you did, if the null hypothesis is correct.

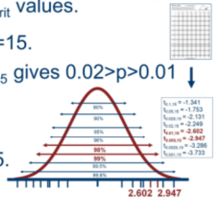
The p value of a test is the probability that the value you see could arise due to sampling error if H_0 is true.

If p value small (~0.05), reject H_0
If p value not small, fail to reject H_0

THE MOST USEFUL CONCEPT IN STATISTICS

The t test practical procedure

- Create a null hypothesis and alternative hypothesis:
  H_0: $\mu=\mu_0$ and H_A: $\mu\neq\mu_0$ $\quad t_{calc}=\dfrac{\bar{x}-\mu_0}{SE}$
- Calculate t_{calc} and compare t_{calc} to various t_{crit} values.
- Determine the p value - e.g., t_{calc}=**2.8** for df=15.
  Use table: $t_{0.01,15}$=2.602<**2.8**<2.947=$t_{0.005,15}$ gives 0.02>p>0.01
  Use computer: calculation gives p=0.013
- Use the small p value to "reject H_0"
  H_0: $\mu=\mu_0$ not consistent with p=0.013<0.05.
  H_A: $\mu\neq\mu_0$ is consistent with p=0.013<0.05.



Caution about one vs two-tailed tests

Doing a one-tailed test allows you to reject H_0 easier (i.e., less difference between $\bar{x}$ and μ_0), so you must be careful.

Only do a one-tailed test under 2 conditions:
1. You only care about one direction
2. You have an *a priori* reason to test in only one direction

You **CANNOT** look at data to choose the direction.

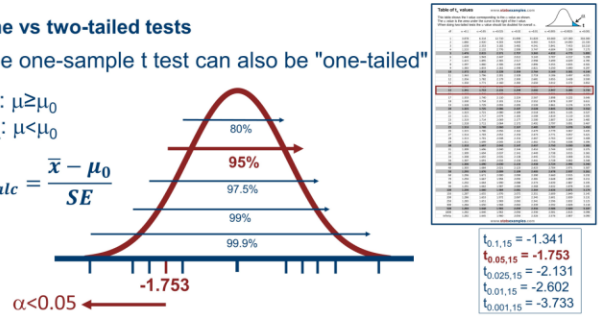
This leads to increased type I errors (i.e., reject a true H_0)

One vs two-tailed tests

The one-sample t test can also be "one-tailed"

H_0: $\mu\geq\mu_0$
H_A: $\mu<\mu_0$

$$t_{calc}=\frac{\bar{x}-\mu_0}{SE}$$



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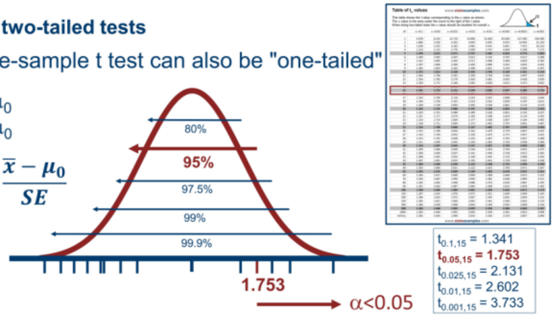
One vs two-tailed tests

The one-sample t test can also be "one-tailed"

H_0: $\mu\leq\mu_0$
H_A: $\mu>\mu_0$

$$t_{calc}=\frac{\bar{x}-\mu_0}{SE}$$



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Statistically significant

The use of p=0.05 (i.e., 5%) as a threshold for deciding to reject null hypothesis is arbitrary, but is the standard.

Statistically significant: A test has returned a p value less than the threshold and the null hypothesis has been rejected.

- If a sample mean of 18 is *significantly different* from μ_0=20, we reject the null hypothesis that the population mean is 20 (i.e., the mean isn't 20).
- If a sample mean of 18 is *not significantly different* from μ_0=20, we fail to reject the null hypothesis that the population mean is 20 (i.e., no evidence it isn't 20).