Introduction to probability

Odds are, you'll find this useful

sample space / Event

Probability fundamentals

We consider the probabilities of events within sample spaces of possible outcomes.

Event: the outcomes we focus on.
Sample space: the set of all possible outcomes.

p(A): the "probability of event A"

sample space / Event / p(event)?

Probability fundamentals

We also consider probabilities of events other than the focal event.

i.e., the probability of the event not being A

Probability of not A represented by:
p(¬A), p(~A), p(A'), p(Ā), p(A^C)

Let's use this one (but remember these if you see them)

sample space / Event / p(not event)?

Sample space = bowl of M&Ms, Event = picking an M&M

e.g., 20 red, 20 yellow, 30 brown, 30 green.

bowl of M&Ms / Event — p(red)=20/100=0.2
bowl of M&Ms / Event — p(¬red)=80/100=0.8

p(purple)=0/100=0
p(M&M)=100/100=1

Some probability rules

p(impossible event) = 0
p(certain event) = 1
} Therefore 0 ≤ p(A) ≤ 1

p(A) + p(¬A) = 1, this is the **Complementation rule**

ADDITION RULES

26/52 = 0.5 — Clubs 13, Hearts 13
8/52 = 0.154 — Jacks 4, Twos 4
16/52 = **0.308** — Jacks 3, (1), Hearts 12

General addition rule:
If A and B *are not mutually exclusive* events,
then p(A or B) = p(A) + p(B) - p(A & B)
p(Jacks or Hearts) = 4/52 + 13/52 - 1/52 = 16/52 = **0.308**

ADDITION RULES

26/52 = **0.5** — Clubs 13, Hearts 13
8/52 = **0.154** — Jacks 4, Twos 4
16/52 = 0.308 — Jacks 3, (1), Hearts 12

Special addition rule:
If A and B *are mutually exclusive* events, p(A or B) = p(A) + p(B)
p(Clubs or Hearts) = 13/52 + 13/52 = 26/52 = **0.5**
p(Jacks or Twos) = 4/52 + 4/52 = 8/52 = **0.154**

e.g., drawing cards from a deck

Drawing a Heart and a Club - mutually exclusive events.
Drawing a Jack and a Two - mutually exclusive events.
Drawing a Heart and a Jack - not mutually exclusive events.

Clubs 13, Hearts 13 | Jacks 4, Twos 4 | Jacks 3, (1), Hearts 12

The complementation rule works because A and ¬A are **mutually exclusive**, an event cannot be both.

e.g., an M&M can't be red and not red.

But, not all events are mutually exclusive.

e.g., drawing cards from a deck.

not red / red

Drawing a heart and a club - mutually exclusive events.
Drawing a heart and a jack - not mutually exclusive events.

Sample space = bowl of M&Ms, Event = picking red M&M

e.g., 20 red, 20 yellow, 30 brown, 30 green.

p(red)=20/100=**0.2**, p(¬red)=80/100=**0.8**

Using the complementation rule:

p(red) + p(¬red) = 1
p(¬red) = 1 - p(red)
p(¬red) = 1 - **0.2** = **0.8**

80 / 20

This example was easy, but this rule allows us to get a hard probability if the complement is easy.

ADDITION RULE APPLICATIONS

p(A or B) = p(A) + p(B) - p(A & B) **(general addition rule)**

but if A and B are mutually exclusive, p(A & B) = 0

p(A or B) = p(A) + p(B) **(special addition rule)**

► We can use these equations to solve for difficult probabilities when we have the others.

► We can determine if events are *mutually exclusive* by separately measuring and then comparing p(A or B), p(A), and p(B).

PROBABILITY SO FAR

p(impossible event) = 0
p(certain event) = 1
For any event A, 0 ≤ p(A) ≤ 1
p(A) + p(¬A) = 1
p(A or B) = p(A) + p(B) - p(A & B)
p(A or B) = p(A) + p(B) if A and B are mutually exclusive

What about an equation for p(A **&** B)?

sample space / Event

MULTIPLICATION RULES

p(A & B) = p(A) x p(B|A)

The probability of B when A is true
"probability of B given A"
This is a **conditional probability**

e.g., for a deck of cards:
A=Club, B=Heart, p(B|A)=0

A=Jack, B=Heart, p(B|A)=1/4
A=Heart, B=Jack, p(B|A)=1/13

Clubs 13, Hearts 13 | Jacks 3, (1), Hearts 12

MULTIPLICATION RULES

e.g., p(A & B) = p(A) x p(B|A)
A: Jack, B: Hearts
p(Jack & Hearts) = 4/52 x 1/4 = 1/52

Jacks 3, (1), Hearts 12

But for this example p(B|A)=p(B):
p(B|A) = p(Hearts|Jack) = 1/4
p(B) = p(Hearts) = 13/52 = 1/4

If P(B) = P(B|A), we say that events A and B are **independent**

MULTIPLICATION RULES

General multiplication rule:
p(A & B) = p(A) x p(B|A)

Special multiplication rule:
p(A & B) = p(A) x p(B) when events A and B are independent

However, events A and B are not always independent.
When this happens, we use a **probability tree** diagram to calculate probabilities.

p(impossible event) = 0
p(certain event) = 1
For any event A, 0 ≤ p(A) ≤ 1
p(A) + p(¬A) = 1

sample space / Event

p(A) — p(B|A) p(A & B); p(¬B|A) p(A & ¬B)
p(¬A) — p(B|¬A) p(¬A & B); p(¬B|¬A) p(¬A & ¬B)

p(A or B) = p(A) + p(B) - p(A & B)
p(A or B) = p(A) + p(B) if A and B are **mutually exclusive**.

p(A & B) = p(A) x p(B|A)
p(A & B) = p(A) x p(B) when events A and B are **independent**.

p(A|B) = (p(A) x p(B|A))/p(B) via **Bayes theorem**.

BAYES THEOREM APPLICATIONS

$$p(A|B) = \frac{p(A) \times p(B|A)}{p(B)}$$

► We can use this equation to solve for conditional probabilities, which are often hard to do.

e.g., randomly test people for a rare disease (**2%** rate, event **A**) with a test that has **1%** false positive and negative rates (positive test is event **B**). What is probability that a random person who tests positive has the disease, p(A|B)?

$$p(A|B) = \frac{p(A) \times p(B|A)}{p(B)} = \frac{(0.02) \times (0.99)}{(0.02 \times 0.99 + 0.98 \times 0.01)} = \frac{0.0198}{0.0296} = 0.67$$

BAYES THEOREM

Since p(A & B)=p(B & A)
and p(A & B) = p(A) x p(B|A)
and p(B & A) = p(B) x p(A|B)

Therefore: p(A) x p(B|A) = p(B) x p(A|B)
and dividing both sides by p(B) gives us **Bayes theorem**:

$$p(A|B) = \frac{p(A) \times p(B|A)}{p(B)}$$

MULTIPLICATION RULE APPLICATIONS

p(A & B) = p(A) x p(B|A) **(general multiplication rule)**

but if A and B are independent, p(B|A) = p(B)

p(A & B) = p(A) x p(B) **(special multiplication rule)**

► We can use these equations to solve for difficult probabilities when we have the others.

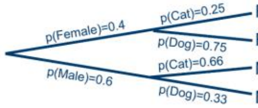
► We can determine if events are *independent* by separately measuring and then comparing p(A & B), p(A), and p(B).

PROBABILITY TREES

When events A and B are not independent.

	Female	Male
Cat	10	40
Dog	30	20

e.g., probability of choosing an animal and it is male and a cat
p(Male & Cat) = 40/100 = **0.4**
p(Male) x p(Cat) = (60/100) x (50/100) = 0.6 x 0.5 = 0.30



Female & Cat, 0.4 x 0.25 = 0.10
Female & Dog, 0.4 x 0.75 = 0.30
Male & Cat, 0.6 x 0.66 = **0.40**
Male & Dog, 0.6 x 0.33 = 0.20

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